Performance Assessment Task

Adding Numbers Grade 3

This task challenges a student to use knowledge of place value to solve a problem using a variety of strategies. The student must demonstrate the ability to make sense of and use someone else's strategy and evaluate the accuracy of the strategy based on calculations and/or understanding and knowledge of place value. A student must be able to calculate accurately using addition, subtraction and multiplication within grade-appropriate parameters.

Common Core State Standards Math - Content Standards

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic. 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3 Multiply one- digit whole numbers by multiples of 10 in the range 10-90 (e.g. 9×80 , 5×60) using strategies based on place value and properties of operations.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5×9 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2007	9	4	58%

Adding Numbers

This problem gives you the chance to:

· work with different methods of adding

There are many different ways to add numbers.

Here is one way to add the numbers 55 and 58.

1. Below are some different ways of adding 55 and 58. Most are correct but some are wrong!

If you think that a statement is correct, draw a ring around the word Correct.

If you think that a statement is wrong draw a ring around the word Wrong

Under each statement show why you think that it is correct or wrong.

a. Double 50 then add 8 then add 5.

Correct Wrong

b. Start with 58 then add 50 then add 5.

Correct Wrong

C.	Double 58 then subtract 3.	Correct	Wrong
d.	Start with 55 then add 60 then subtract 2.	Correct	Wrong
e.	Add 5 and 8 then add 100.	Correct	Wrong
f.	Add 50 and 60 then subtract 5 and subtract 2.	Correct	Wrong

Tas	sk 3: Adding Numbers		Ru	bric
• wo	core elements of performance required by this task are: rk with different methods of adding d on these, credit for specific aspects of performance should b		points	section points
a	Rings correct answer: Correct			
	Shows calculation such as: $50 + 50 = 100$	00 + 8 + 5 = 113	1	
b	Rings correct answer: Correct			
	Shows calculation such as: $58 + 50 = 108$	08 + 5 = 113	1	
c	Rings correct answer: Wrong and			
	Shows calculation such as: $55 + 55 = 110$	10 + 8 = 118	2	
	Gives correct answer such as: Only 3 should be added, not 8.			
d	Rings correct answer: Correct			
	Shows calculation such as: $58 + 58 = 116$	16 - 3 = 113	1	
e	Rings correct answer: Correct			
	Shows calculation such as: $55 + 60 = 115$	15 - 2 = 113	1	
f	Rings correct answer: Correct			
	Shows calculation such as: $5 + 8 = 13$	3 + 100 = 113	1	
g	Rings correct answer: Wrong and			
	Shows calculation such as: $50 + 60 = 110$	10 - 5 - 2 = 103	2	
				9
		Total Points		9

55

Adding Numbers

Work the task and look at the rubric. What do you think are big mathematical ideas being assessed in this task? What does a student need to understand to work these problems?

Think about some basic number properties:

Equality Property

If a = b, then a + c = b + c.

Commutative Property

In addition, a + b = b + a. While subtraction is not commutative, a subtraction problem can be rewritten as addition to use Commutative Property. For example: 8 - 2 = 6

$$8 + ^{-}2 = 6$$
 and $^{-}2 + 8 = 6$.

Associative Property

In addition, (a +b) + c = a + (b + c)

By combining these properties with substitution, numbers can be decomposed to simplify computations. This is what allows the strategies in Adding Numbers to work or "not work". A major piece of mathematics is being able to justify or explain why something works, why it is true. At this grade level, students don't need to have the formal proofs using these properties, but they should have the habit of mind to justify a strategy by showing that it will or will not give an equivalent answer to the original problem. Look at your student work.

What percentage of your students did not show any calculations?	

How do you help establish classrooms norms around justification or explaining their thinking? Are students encouraged to show their work or provide labels on computations?

Now think about the different demands of the task. Look at work for b and f, adding oneand two- digit numbers. Are students able to align the numbers and think about place value or are they getting answers like 613, 181, or 230?

Now look at student work on doubling: part a, c, and d. How many of your students:

Doubling using	Doubling using	Doubling on only 1	No Doubling
addition	multiplication	or 2 parts	

Why do you think students may have been confused by this? Why might their thinking be inconsistent?

some of the st and do the cor	rudents have wo rategies being p nputations. Ye look at student	resented, all stu t, some students	dents should be had difficulty	e able to read the with the parts as	e directions sking for
subtraction?	y at the work on w to break it apa			think about the	consecutive
Did they atten	npt to do it as it	were column ac	ldition?		
·	oine the 5+2 firs				
	answers. How i			NT 1	0.1
103	117	104	113	Numbers >	Other

What might be your next steps to help students think about the consecutive subtractions? How could you set up a task that would confront these misconceptions? How would you manage the discussion?

Do students in your class have opportunities to do activities like number talks, where they decompose and recompose numbers? How does this help develop their computational fluency? How does this help them to understand the number system, place value, and number properties?

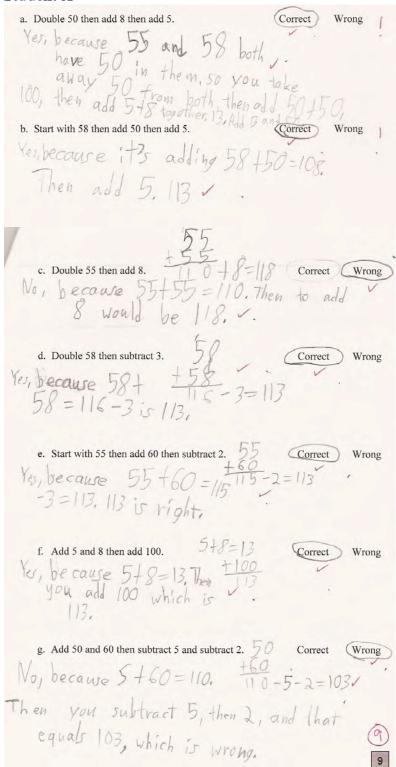
Thinking about the student work, what concerned or surprised you about their responses? What might be your next steps in working with them? What might you do differently next year?

120

Looking at Student Work on Adding Numbers

Student A justifies the answer by giving verbal descriptions and calculations. Although notationally, the student uses 2 subtractions the verbal description shows that the student is thinking about each part separately.

Student A



Student B seems to look at the directions first and think, "How can I get this back to the original numbers 55+58. For example, look at how the student makes conscious choices about what to do first in problems a, b, and d. Also note that in part g the student chose to write the subtraction as two separate steps.

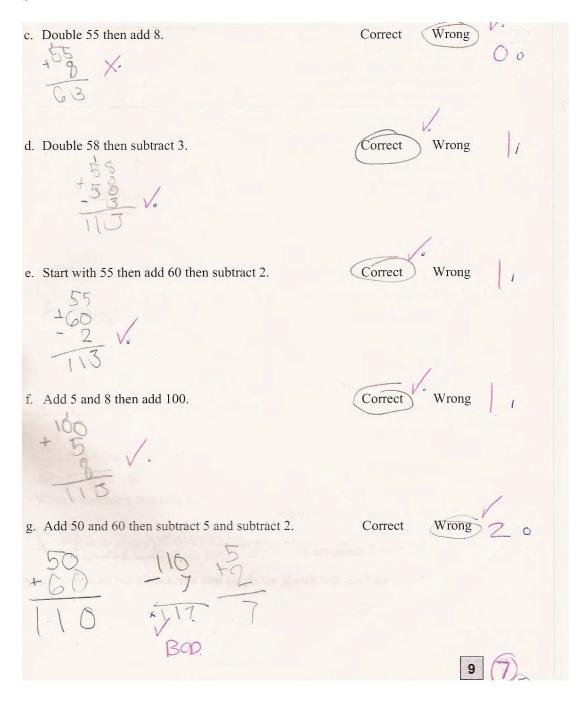
a. Double 50 then add 8 then add 5. $50 + 8 = 58$ $50 + 5 = 55$ 13	Correct Wrong
b. Start with 58 then add 50 then add 5. 50 +5 = 55 +58	Correct Wrong
c. Double 55 then add 8. $+\frac{55}{55} + \frac{110}{118}$	Correct Wrong 2
d. Double 58 then subtract 3. -58 +59 +58 +58 +58 +58	Correct Wrong
e. Start with 55 then add 60 then subtract 2. $ \begin{array}{c} $	Correct Wrong
f. Add 5 and 8 then add 100. $\frac{5}{13} + \frac{13}{113} = $	Correct Wrong
g. Add 50 and 60 then subtract 5 and subtract 2. $\frac{50}{60} - \frac{105}{5} = \frac{2}{105}$	Correct Wrong 2.

Student C is one of the few students who thought about doubling as multiplication. Notice that the student mentally combines the 5 and 2 before performing the subtraction.

c. Double 55 then add 8.		Correct	Wrong 2
d. Double 58 then subtract	2 2 118/	Correct	Wrong /
e. Start with 55 then add	60 then subtract 2.	Correct	Wrong /
f. Add 5 and 8 then add	100.	Correct	Wrong .
g. Add 50 and 60 then su	100 113V	Correct	Wrong 2
g. Add 30 and 00 then su	50 60 10	Concer	(a) (a)
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Most students understood the idea of doubling as adding the same number together twice. However in trying to think about several ideas at once, which numbers, which operations, that idea of doubling was not consistent throughout their work. In part g, Student D combines the 5 + 2 to get 7, before subtracting. However, again the student loses focus and writes subtraction but performs addition. What are some reasons for these types of errors? Are there activities or experiences that will help students build their competence to work with several directions?

Student D



Student E does not understand doubling. The student uses addition even for the subtractions in d and g. While the student doesn't align numbers in some of the problems, his mental process allows him to get a correct solution.

a. Double 50 then add 8 then add 5.	Correct	Wrong O
50		
+51		
b. Start with 58 then add 50 then add 5.	Correct	Wrong Oo
58		
50		
15		
133 ×		

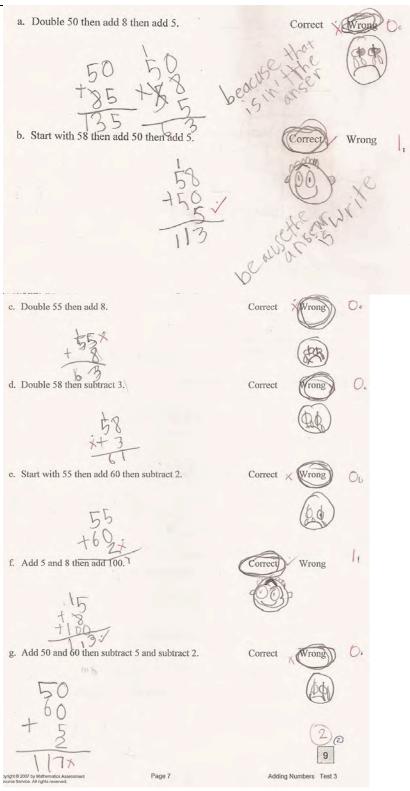
Student E, part 2

Student E, part 2				
c. Double 55 then add 8.		Correct	Wrong O o	
+855			×	
d. Double 58 then subtract	3.	Correct	Wrong	
58 61 ×			×	
e. Start with 55 then add 60) then subtract 2.	Correct	(Wrong)	
55 +60				
f. Add 5 and 8 then add 10	0.	Correct	Wrong	
100				
g. Add 50 and 60 then subtr	ract 5 and subtract 2.	Correct	Wrong	
60 50				
+2			9	
TITX		V. Maria		
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Student F also struggles with the issue of subtraction. In part d she does both. While she writes subtraction in part g, she tries to do both subtractions in one step. In reality she has added the five and only subtracted the final digit 2.

c. Double 55 then add 8.		Correct	Wrong	0
+ 50			×	
d. Double 58 then subtract	3.	Correct	Wrong	0
$\frac{-55}{52}$ $\sqrt{53}$ $\frac{-3}{56}$	X	1/		,
e. Start with 55 then add 60	then subtract 2.	Correct	Wrong	
55 +60 115 113				1.
f. Add 5 and 8 then add 100).	Correct	Wrong	
100				1-
g. Add 50 and 60 then subtr	act 5 and subtract 2.	Correct	Wrong	
+60				0.
-110				3
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Student G combines the 8 and 5 to make 85 before adding. Although the student is able to self correct, this mistake happed with other students. Again, like the previous students G does not use subtraction.



Student H chooses to do the subtraction before the addition in part 2. Wrong g. Add 50 and 60 then subtract 5 and subtract 2. Correct

Student I has several problems with place value. The student does not align numbers by value. Notice again the difficulties when trying to subtract twice in part g.

a.	Double 50) then add	8 then	add 5.

Correct Wrong

b. Start with 58 then add 50 then add 5.

Correct

Wrong ():

Student I, part 2

Student 1, part 2	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
c. Double 55 then add 8.	Correct Wrong 2 2
455 255 410	
d. Double 58 then subtract 3.	Correct Wrong 0 0
+58	
e. Start with 55 then add 60 then subtract 2.	Correct Wrong O
+55 60 -115	
f. Add 5 and 8 then add 100.	Correct Wrong O o
+100	
g. Add 50 and 60 then subtract 5 and subtract 2.	Correct Wrong O o
+50 =119	Subst
110 113x	(1) (3) (9)
	Adding Numbers Test 3

Student K also has difficulties with place value. Can you figure out how she got her answers for d and g?

c. Double 55 then add 8.	55 8 x	Correct Wrong	
	63×		0.
d. Double 58 then subtract 3	3.	Correct	
e. Start with 55 then add 60	58 58 + 3 = 73 × then subtract 2.	Correct Vrong	O ·Place
f. Add 5 and 8 then add 100	55 60* 2-x*	Correct Wrong	0.
100	3+ /v		
g. Add 50 and 60 then subtra	act 5 and subtract 2.	Correct Wrong) 6.
5 QO opyright © 2007 by Mathematics Assessment	Page 7	Adding Numbers Tas	2

Student Task	Work with different methods of adding. Determine if the method gives	
	the correct solution.	
Core Idea 1	Understand numbers, ways of representing numbers, relationships	
Number	among numbers and number systems.	
Properties	 Develop a sense of whole numbers and represent and use them in flexible ways including relating, composing, and decomposing numbers. Understand the place-value structure of the base-ten number system including being able to represent and compare whole numbers. 	
Core Idea 2	Understand the meanings of operations and how they relate to each	
Number	other, make reasonable estimates, and compute fluently.	
Operations	 Develop fluency in adding and subtracting whole numbers. 	

Mathematics in this task:

- Ability to use place value to add and subtract one- and two- digit numbers
- Ability to use properties of numbers to simplify computations
- Ability to make and to justify conclusions using computation

Based on teacher observations, this is what third graders know and are able to do:

- Add and subtract well
- Double
- Some students knew that if they could transform the numbers back to 50 + 58 they didn't need to do any more work

Areas of difficulty for third graders:

- Follow all the directions, they often left off the final step
- Choosing correct or wrong (Some students correctly calculated the 113, but circled wrong)
- Showing their work (18% of the students did not show any calculations)
- Aligning numbers to maintain place value
- Breaking up the subtraction in part g to 2-steps

Task 3 - Adding Numbers

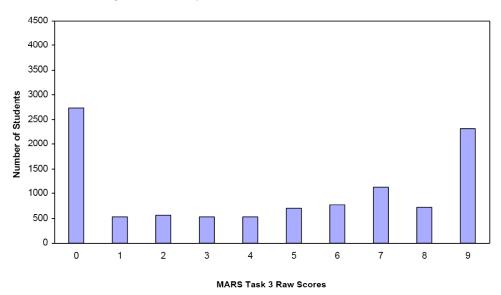
Mean: 4.54

Table 17: Frequency Distribution of MARS Test Task 3, Grade 3

StdDev: 3.54

Task 3	Student	% at or	% at or
Scores	Count	below	above
0	2741	26.1%	100.0%
1	532	31.2%	73.9%
2	568	36.6%	68.8%
3	529	41.7%	63.4%
4	527	46.7%	58.3%
5	703	53.4%	53.3%
6	760	60.7%	46.6%
7	1114	71.3%	39.3%
8	708	78.0%	28.7%
9	2304	100.0%	22.0%

Figure 26: Bar Graph of MARS Test Task 3 Raw Scores, Grade 3



The maximum score available for this task is 9 points.

The minimum score needed for a level 3 response, meeting standards, is 4 points.

Many students, 74% could do part b of the task, adding three numbers. More than half the students, 63%, could do part b and f involving only addition and part e involving a subtraction. About 60% could also do one of the doubling tasks, usually a which only required addition. Almost 40% of the students could do all parts of the task except part g, which required two consecutive subtractions. 22% of the students could meet all the demands, including understanding place value, adding and subtracting one and two digit numbers including tow consecutive subtractions. 26% of the students scored no points on the task. 93% of those students marked the correct or wrong, but showed no computation to support their answer.

Adding Numbers

Points	Understandings	Misunderstandings
0	93% of the students with this	Students who attempted the task, showed
	score attempted the task.	no computations to support their answers.
		The answers may or may not have been
		correct.
		Figures below are for those students in the
		sample who showed calculations.
2	Students could complete part b	6% of the students calculated 113 for part f,
	and f, which required only	but circled wrong. Almost 10% made
	addition.	mistakes based on place value for part f,
	Students could do addition	with answers like 181, 230, and even 1700.
4		16% of the students forgot to double in part a. 8% added instead of subtracted the 2 in
	problems b and d, a doubling problem such as a which is all	part 5. 4% ignored the 2 in part e, giving
	addition, and part e an addition	themselves an answer of 115. another 4%
	with no doubling and a	calculated 113, but still circled wrong.
	subtraction.	outsured 110, out sum officer wrong.
7	Students could double, add, and	Doing two consecutive subtractions was
	subtract accurately. 71% of the	hard for students to think about. Only 33%
	students who attempted part 1,c,	of the students actually wrote it as two
	or d were able to double by	separate calculations. An additional 10%
	adding a number twice. Only	were able to combine the 5 and 2, so they
	7% used multiplying by 2 as a	could subtract once by 7. 25% tried to set
	strategy for doubling.	up the problem like an addition problem
		with both subtractions at once. While
		some students were able to still do the
		calculation correctly, this led to many
		problems for students. In part g, 10% of the students added both numbers (117), 13%
		added the 5 and subtracted only the 2
		(113).
9	Students could decompose	(/:
	numbers and perform	
	calculations using doubling,	
	addition and subtraction to	
	verify whether a strategy would	
	yield a solution of 113.	

Implications for Instruction

Students need to develop a deeper understanding of place value. Experiences like number talks helps them focus on the value of the digits and reason about the size of the numbers. In the process of composing and decomposing numbers they are also developing computational fluency and a working knowledge of the number system and properties of numbers. Students need ample experiences where they set up problems for themselves. This allows misunderstandings about aligning numbers and place value to surface, so that they can be discussed explicitly, and students can confront the logic behind the algorithms and see the reasoning for "lining up" the numbers. In seeing that adding something to make a friendly number requires a later subtraction, students are developing a basic algebraic understanding about "doing and undoing" or inverse operations. This sense of compensating for each action also helps them to think deeply about equivalency.

Students need help with subtraction. They should know a variety of strategies for thinking about addition and subtraction of 2-digit numbers. An understanding of part/part/whole relationships in addition and subtraction allows students to be more flexible in finding missing parts in addition and subtraction and to see the connection between the two. While many students have not encountered situations directly asking for consecutive subtractions, students should be familiar enough with the operation to think that 110

- 5

_____2 does not look right. They should have some sort of "self-talk" about this looks strange or why haven't I done this before, what could this mean? So even in a problem-solving mode they should have some information to help them think about this situation.

Students should also understand the meaning of doubling a number. While it is useful to understand doubling as adding the same number to itself, students at this grade should start to move to multiplicative thinking or doubling as two times a number.

Finally, memory or attention to detail is a skill that can be developed through practice. Students need opportunity to hold information in their head and work through multiple steps or notice multiple attributes. As students move through the grades, they need tasks with increasing chains of reasoning and higher cognitive demands for justification.

Ideas for Action Research Number Talks – consecutive subtractions

Set up a problem for students, like:

Add 40 and 30 then subtract 6 and subtract 2.

Have students use white boards or large white paper with markers that they can hold up to make sharing easy. Using white boards or erasable material is good because students can revise their work without the stigma of crossed out work. Ask students to compare their answers and strategies with a partner. Poll the class: How many of you had the same answer as your partner? Do you think there can be more than one answer to this problem? Have them hold up their answers. Now put some of those answers on the board:

150 74 62 78 152 *other*

Ask students to discuss which one is correct by sharing strategies and trying to make a convincing argument. Also try to ask questions that get students to think about why their strategy might not work.

If no one in the class suggests 70 - 8, say a student in another class tried it. Do you think it works? Why? Or Why not? Does it make sense?

What other ideas for Number Talks does the student work suggest. Plan one of your own.